The Visual Side of Learning

Teacher Note

Contrary to what many people believe, students frequently do not use definitions of concepts in their thinking. Instead, they use concept images: a combination of all the mental pictures and properties that they have associated with a concept.

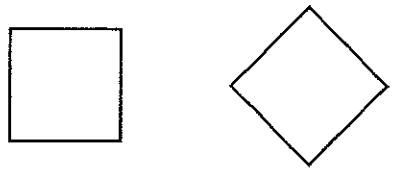
Illustrations used in instruction have a strong influence on these concept images. For example, if students see figures only in certain "standard" positions, they may assume that the figures occur only in those positions (as illustrated by the common misconception about squares, discussed below).

Even when students learn correct, standard verbal descriptions or definitions of a concept, their concept images, influenced by their visual experience, tend to rule their thinking. Therefore, they need to combine each verbal description they encounter with a wide variety of visual examples.

It is important to provide the variety of visual examples both when a concept is introduced and later, to help students construct meaningful verbal descriptions and definitions from the examples. If verbal descriptions are to be of real use, students need to understand them thoroughly. That is why the activities in these investigations are set up so that students construct their definitions based on many different images.

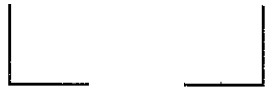
For example, many students may start the unit with limited ideas about triangles. You can be instrumental in encouraging them to understand which characteristics are relevant to the definition of a triangle (three sides, closed figure), and which characteristics are irrelevant (size and orientation, for example), and to synthesize the resulting visual images and verbal definition.

In one classroom, a teacher drew two squares on the overhead, oriented like these.

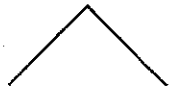


When asked if both were squares, several students said no—that one was a diamond. After some discussion, the teacher pointed out that nothing in the rules for squares says that they must have horizontal and vertical sides.

The teacher then drew two right angles, oriented as below.



After students established that their right-angle tester (a plastic square) worked for both angles, the teacher drew a third right angle:



This time, after a little discussion, a student volunteered that "nothing in the rules for right angles says they have to sit flat." The teacher acknowledged this application of the definition.

Discussions like this help students to visualize more possibilities for squares and right angles. They also help them consider and apply the definitions of the terms, recognizing that what the definitions do *not* say can be as important as what they do say.

Teacher Note

Classification of Triangles and Quadrilaterals

Classification systems help us to organize the world into categories that often are hierarchical and overlapping. For example, a person might live in a particular town, which is in a particular state, which is in the United States, which is in North America, which is in the Western Hemisphere. If I say that I live in that town, you know that I live in all the other places that include the town; for example, everyone who lives in Cleveland also lives in Ohio, and so on. In a hierarchical classification system, we can't make the same kind of assumptions in reverse; it is not true that all people who live in Ohio live in Cleveland.

We use a hierarchical classification system to sort geometric figures. The activities in this unit help clarify the classification of triangles and quadrilaterals.

Triangles

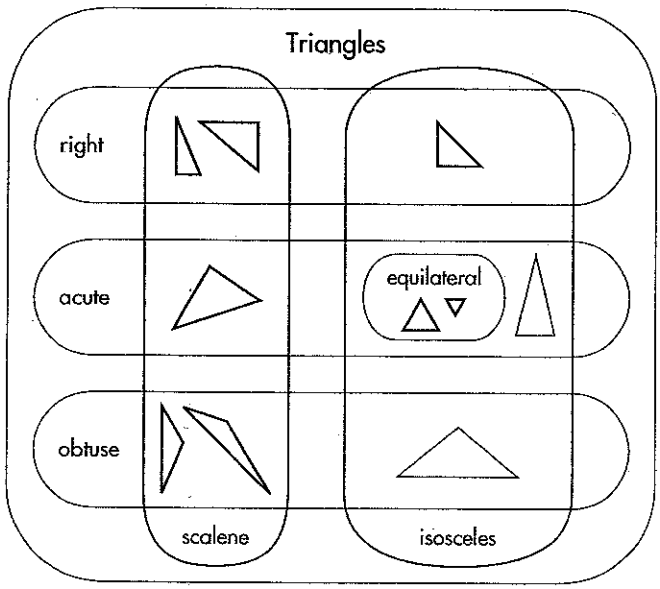
There are two ways to classify triangles, by their angles and by their sides. Classified by their *angles,* triangles are right (one 90° angle), acute (all angles smaller than 90°), and obtuse (one angle greater than 90°). These categories are illustrated by the horizontal loops in the diagram. Classified by their *sides,* triangles are scalene (no sides the same length), isosceles (at least two sides the same length), or equilateral (all sides the same length).

Consider two triangles:

Triangle A has two sides the same length and a 90° angle, so it is both right and isosceles: a right isosceles triangle. Such double classifications take some time to get used to.

There are also more complex relationships: Triangle B is acute because it has three acute angles; it is equilateral because it has three equal sides; and that means it is also isosceles, because at least two of its sides are equal. That is why it lies inside three loops. So Triangle B is an *equilateral* triangle, all of which are members of the isosceles triangle family, which are members of the triangle family, which are members of the polygon family.

We usually speak of an object by its most restrictive category: If a triangle has three equal sides, we call it *equilateral* rather than isosceles or just a triangle. In providing illustrations, we do the opposite: We provide the least restrictive example. To illustrate an isosceles triangle, we draw one with two equal sides, not three; for a general triangle, we draw a scalene triangle without a right angle.



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Quadrilaterals

There are two ways to classify quadrilaterals. The first asks if they have parallel sides. *Trapezoids* have at least one pair of parallel sides. *Parallelograms* have two pairs of parallel sides.

The second way of classifying quadrilaterals concerns the lengths of their sides. *Kites* and *chevrons* have two pairs of equal adjacent sides. *Isosceles trapezoids* have one pair of equal opposite sides. *Parallelograms* have two pairs of equal opposite sides. *Rhombuses* (or rhombi) are members of the parallelogram family that have all four sides equal.

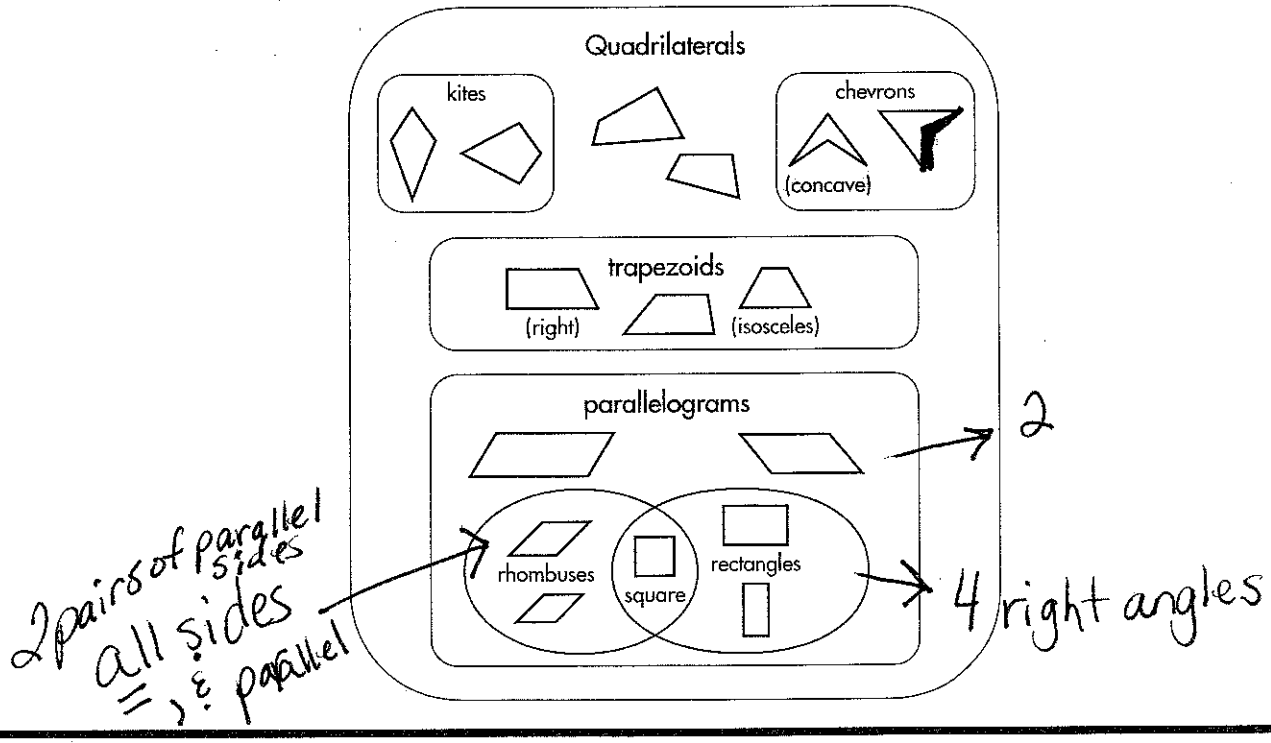
The angles are what make rectangles special. Rectangles are members of the parallelogram family with four equal angles. Squares, then, are in many families, including rectangle, rhombus, and parallelogram.

The diagram below shows the complicated hierarchical classifications of quadrilaterals, which include trapezoids and parallelograms.

Trapezoids can be further classified into isosceles (one pair of opposite sides equal) or not isosceles, and right (one right angle) or not. Isosceles concave quadrilaterals are commonly known as chevrons. Kites are another special kind of quadrilateral.

Such traditional classifications are just one useful way to sort geometric figures. We could just as well declare that rectangles *cannot* have all equal sides, and then squares would not be in the family of rectangles. Students often prefer this partitioning way of classifying. Only with time will they come to see the advantages of hierarchical classification—for example, economical definitions and logical inference (if you know a square is a rectangle, you know it has all the properties of rectangles).

At this age, students will benefit from thinking and communicating about the properties of polygons, but they need not have the whole classification system in mind.



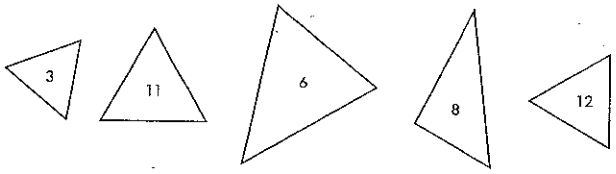
DIALOGUE BOX

Are All Three-Sided Polygons Triangles?

As this class discusses categories for the triangles on the Guess My Rule Cards, some students are confused about whether *all* three-sided polygons should be called triangles.

We're looking for categories for sorting three-sided polygons. So far we have shapes with two equal sides, shapes with all equal sides, shapes with all unequal sides, and shapes that if you had another shape just like it, you could make a square or rectangle. What else?

Robby: How about shapes that look like triangles. Examples are 3, 11, 6, 8, and 12. Not 9.



What must a figure have to look like a triangle?

Robby: Three equal sides. Like one would be left side 15 cm, right side 15 cm, and the bottom connecting the two 15 cm.

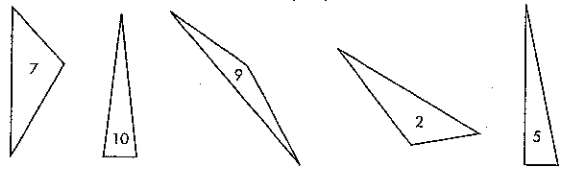
Robby feels the other three-sided polygons aren't triangles. What do the rest of you think? Does a polygon have to have three equal sides to be called a triangle?

Yu-Wei: The shape of 9 is a stretched triangle, so it's not a triangle.

Julie: I disagree, because other ones that are sort of stretched like 6 and 8 *are* triangles.

Robby, which ones do you feel are not triangles?

Robby: I think 7, 10, 9, 2, and 5 aren't.



[There is a lot of discussion at this point, and the teacher asks to hear more about what makes a shape a triangle.]

Help! What *is* a definition of a triangle?

Lindsay: It has to have three angles. You know that because *tri*-angle; *tri* means "three."

If *triangle* means three angles, where do the sides fit in?

Matt: Anything with three sides is a triangle.

Manuel: I agree with three sides, but I think they all have to be equal.

Leon: Then how come they have triangles called isosceles and scalene?

[Class ends with the question unresolved. Discussion continues the next day.]

We ended math yesterday with a big question: Are all three-sided polygons triangles? What did you decide?

Cara: I wrote: "I think a triangle is a shape that has three sides and three corners. I think that triangles don't have to have the same length in the sides."

Robby: I made a category called "Triangles," and I wrote, "A triangle should have even sides and three corners."

So what about three-sided shapes that don't have even sides? What would you call those?

Robby: I don't know.

Antonio: I used to think like Robby, but now I think any three-sided polygon is a triangle. I didn't know what to call those other shapes except triangles.

Julie: I agree. I think a triangle's something with three sides no matter how long they are.

Manuel: Triangles can have different sides, and then have other names that mean they're not equal, like *right* or *isosceles* triangles.

It's important that we're coming to agreement about these terms as a class.

DIALOGUE BOX

Are Squares Rectangles?

This class is discussing quadrilaterals when a controversy arises: Is a square a rectangle? Is a rectangle a square? The teacher chooses one shape, a square, on which to center the discussion.

What is this *[holding up a square]*: a square, a rectangle, or both?

Yu-Wei: It's a square.

Becky: But a square is a rectangle, isn't it?

Amy Lynn: No, a rectangle is a square, but a square isn't a rectangle.

What do we know has to be true about a square?

Amy Lynn: All 90° angles and even amounts of edges or sides.

So what about this shape? *[The teacher draws a square on the board.]*

Becky: You know what you could call it? An equilateral rectangle.

Becky, can you explain your reasoning? I see a lot of people don't agree.

Becky: A rectangle is four sides that have parallel sides somewhere else in the shape. A square has those things, but instead of two sides being different, they're all the same.

OK, for a square, the sides all have to be equal. Now, could we call a square a rectangle?

Desiree: A rectangle doesn't have all sides equal; it only has two.

Julie: A rectangle has two equal sides. It has to have two equal sides, and a square has four equal sides. But in those four equal sides, a square has the two it needs to be a rectangle.

Amy Lynn: Well, then you should be able to reverse that too.

Julie: But a rectangle can't be a square, because a rectangle has two sides equal and then another two sides equal but different from the other and a square *has* to have four equal sides. Becky's right, a square can be a rectangle.

Amy Lynn: I agree, but a rectangle can be a square because it's just the opposite.

Manuel: A square can always be a rectangle, but only a rectangle with four equal sides can be a square.

Amy Lynn: But then you'd just call it a square.

Manuel: You could call it a square or a rectangle.

Amir: It's like first and last names in families. You can call them square rectangles.

Lindsay: Or an equilateral rectangle, like Becky said.